UN – 177

V Semester B.A./B.Sc. Examination, November/December 2015
(Semester Scheme) (NS)
(2013-14 and Onwards)
MATHEMATICS – VI

Time : 3 Hours Max. Marks : 100

Instruction: *Answer* ***all*** *questions.*

I. Answer **any fifteen** questions. **(15×2=30)**

1) Solve $yz \log z\, dx - zx \log z\, dy + xy\, dz = 0$.

2) Verify the condition for integrability
$(2y - z)\, dx + 4dy - 2dz = 0$

3) Form the partial differential equation by eliminating arbitrary constants $z = (x + a)(y + b)$.

4) Solve $p^2 - q^2 = 1$.

5) Solve Lagrenge's linear equation $xp + yq = z$.

6) Solve $\left(D^2 - 5DD' + 6D'^2\right) z = 0$.

7) Write the Rodrigeres formula for the Legendre polynomials and use it to find $P_2(x)$.

8) Show that $P_n'(1) = \frac{n(n+1)}{2}$.

9) Show that $J_{-n}(x) = (-1)^n J_n(x)$.

10) Write the Bessels differential equation.

11) Prove that $\frac{d}{dx}[x J_1(x)] = x J_0(x)$.

12) Evaluate $\Delta \tan x$.

13) Prove that $\left(E^{1/2} + E^{-1/2}\right)(1+\Delta)^{1/2} = 2 + \Delta$.

14) Estimate the missing term from the table.

x	0	1	2	3	4
y	1	3	9	...	81

15) Write the Lagrenge's inverse interpolation formula.

16) Evaluate $\int_0^1 \frac{dx}{1+x}$ using Trapezoidale rule.

17) What are Empirical and Theoretical models ?

18) In the case of modelling of a projectile motion without air resistance, find the maximum range on the horizontal.

19) Uranium disintegrates at a rate proportional to the amount present at any instant of m_1 and m_2 grams of Uranium are present at times t_1 and t_2 respectively, show that half life period of Uranium is $\frac{(t_1 - t_2)\log 2}{\log\left(\frac{m_1}{m_2}\right)}$.

20) Write the differential equation of the free damped in Modelling Mass-Spring-Dashpot.

II. Answer **any four** questions : **(4×5=20)**

1) Verify the condition for integrability and solve
$2yzdx + zxdy - xy\,(1 + z)dz = 0$.

2) Solve $\frac{dx}{y+z} = \frac{dy}{z+x} = \frac{dz}{x+y}$.

3) Solve $p^2 + q^2 = z^2\,(x + y)$.

4) Solve by Charpits method $z = pq$.

5) Solve $(D^2 - 5DD' + 4D'^2)\,z = \cos(2x + 3y)$

6) Solve one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ given

i) the end points are at zero temperature

ii) the initial and boundary conditions are $u(x,0) = f(x)$, $u(0,t)$ and $u(l, t) = 0\ t \geq 0$

OR

Solve $x^2p^2 + y^2q^2 = z^2$

III. Answer **any three** questions : (3×5=15)

1) Prove that $\frac{1-t^2}{\left(1-2xt+t^2\right)^{3/2}} = \sum_{n=0}^{\infty}(2n+1)\,P_n(x)t^n$.

2) Show that $\int_{-1}^{1} P_m(x)P_n(x)\,dx = 0$ if $m \neq n$.

3) Prove that $xP_n'(x) - P_{n-1}'(x) = nP_n(x)$.

4) Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\ \sin x$.

5) Show that $\frac{d}{dx}\left(J_n^2 + J_{n-1}^2\right) = \frac{2}{x}\left[nJ_n^2 - (n+1)J_{n+1}^2\right]$.

IV. Answer **any four** questions : (4×5=20)

1) Obtain the function whose first difference is $x^3 + 3x^2 + 5x + 12$.

2) By the method of separation of symbols, prove that

$$u_0 + \frac{u_1 x}{1!} + \frac{u_2 x^2}{2!} + \frac{u_3 x^3}{3!} + \ldots = e^x\left[u_0 + x\frac{\Delta u_0}{1!} + x^2\frac{\Delta^2 u_0}{2!} + \ldots\right].$$

3) Evaluate f(1.4) from the following data

x	1	2	3	4	5
f(x)	10	26	58	112	194

4) Find the cubic function from the following table using Newtons divided difference formula.

x	0	1	2	5
f(x)	2	3	12	147

5) Find $\frac{dy}{dx}$ at $x = 1$ from the following table

x	1	2	3	4	5
y	0	6	24	60	120

6) Evaluate $\int_0^1 \frac{x}{1+x^2} dx$ by Simpson's $\frac{3}{8}^{th}$ rule by dividing the interval into three equal parts.

V. Answer **any three** questions : **(3x5=15)**

1) Discuss the classification of mathematical model in detail.
2) In a population of birds, the proportionate birth and the proportionate death rate are both constant, being 0.45 per year and 0.65 per year respectively. Formulate a model of the population and discuss its long term behaviour.
3) Form the differential equation of the free damped motion in the case of Mass Spring-Dashpot and discuss (i) over damped (ii) critically damped cases.
4) A radio active sustance disintegrates at a rate proportional to its mass, when its mass is 10 mgm the rate of disintegration is 0.051 mgm per day. How long will it take for the mass to be reduced from 10 mgm to 5 mgm ?
5) Discuss the modelling of electric circuits (LCR).