V Semester B.A./B.Sc. Examination, November/December 2015 (Semester Scheme) (NS) (2013-14 and Onwards) MATHEMATICS - VI

Time: 3 Hours

Max. Marks: 100

Instruction: Answerall questions.

I. Answer any fifteen questions. (15×2=30)

- Solve yz logz dx zx logz dy + xy dz = 0.
- 2) Verify the condition for integrability (2y - z) dx + 4dy - 2dz = 0
- 3) Form the partial differential equation by eliminating arbitrary constants z = (x + a) (y + b).
 4) Solve p² = q² = 1
- 4) Solve $p^2 q^2 = 1$.
- 5) Solve Lagrenge's linear equation xp + yq = z.
- 6) Solve $(D^2 5DD' + 6D'^2)z = 0$.
- 7) Write the Rodrigeres formula for the Legendre polynomials and use it to find $P_{2}(x)$.
- 8) Show that $P'_{n}(1) = \frac{n(n+1)}{2}$.
- 9) Show that $J_{-n}(x) = (-1)^n J_n(x)$.
- 10) Write the Bessels differential equation.
- 11) Prove that $\frac{d}{dx}[x J_1(x)] = xJ_0(x)$.
- 12) Evaluate ∆tanx.
- 13) Prove that $\left(E^{\frac{1}{2}} + E^{-\frac{1}{2}}\right) (1+\Delta)^{\frac{1}{2}} = 2+\Delta$.
- 14) Estimate the missing term from the table.

	x	0	1	2	3	4
I	у	1	3	9	***	81

- 15) Write the Lagrenge's inverse interpolation formula.
- 16) Evaluate $\int_{0}^{1} \frac{dx}{1+x}$ using Trapezoidale rule.
- 17) What are Empirical and Theoretical models?
- 18) In the case of modelling of a projectile motion without air resistance, find the maximum range on the horizontal.
- 19) Uranium disintegrates at a rate proportional to the amount present at any instant of m1 and m2 grams of Uranium are present at times t1 and t2

respectively, show that half life period of Uranium is $\frac{(t_1-t_2)\log 2}{\log \left(\frac{m_1}{m_2}\right)}$.

- 20) Write the differential equation of the free damped in Modelling Mass-Spring-BMSCW Dashpot.
- II. Answer any four questions:

 $(4 \times 5 = 20)$

- 1) Verify the condition for integrability and solve 2yzdx + zxdy - xy(1 + z)dz = 0.
- 2) Solve $\frac{dx}{v+z} = \frac{dy}{z+x} = \frac{dz}{x+y}$
- 3) Solve $p^2 + q^2 = z^2 (x + y)$.
- 4) Solve by Charpits method z = pq.
- 5) Solve $(D^2 5DD' + 4D'^2)$ z = cos (2x + 3y)
- 6) Solve one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ given
 - i) the end points are at zero temperature
 - ii) the initial and boundary conditions are u(x,0) = f(x), u(0,t) and u(l,t) = 0 $t \ge 0$

Solve $x^2p^2 + v^2q^2 = z^2$

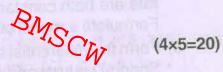
III. Answer any three questions:

 $(3 \times 5 = 15)$

1) Prove that
$$\frac{1-t^2}{\left(1-2xt+t^2\right)^{3/2}} = \sum_{n=0}^{\infty} (2n+1) P_n(x)t^n.$$

- 2) Show that $\int_{-1}^{1} P_m(x) P_n(x) dx = 0$ if $m \neq n$.
- 3) Prove that $xP'_{n}(x)-P'_{n-1}(x)=nP_{n}(x)$.
- 4) Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.
- 5) Show that $\frac{d}{dx} \left(J_n^2 + J_{n-1}^2 \right) = \frac{2}{x} \left[n J_n^2 (n+1) J_{n+1}^2 \right]$.

IV. Answerany four questions:



- 1) Obtain the function whose first difference is $x^3 + 3x^2 + 5x + 12$.
- 2) By the method of separation of symbols, prove that

$$u_0 + \frac{u_1x}{1!} + \frac{u_2x^2}{2!} + \frac{u_3x^3}{3!} + \dots = e^x \left[u_0 + x \frac{\Delta u_0}{1!} + x^2 \frac{\Delta^2 u_0}{2!} + \dots \right].$$

3) Evaluate f(1.4) from the following data

x	1	2	3	4	5
f(x)	10	26	58	112	194

 Find the cubic function from the following table using Newtons divided difference formula.

х	0	1	2	5
f(x)	2	3	12	147



5) Find $\frac{dy}{dx}$ at x = 1 from the following table

×	1	2	3	4	5
У	0	6	24	60	120

6) Evaluate $\int_{0}^{1} \frac{x}{1+x^2} dx$ by Simpson's $\frac{3}{8}^{th}$ rule by dividing the interval into three equal parts. (3×5=15)

V. Answer any three questions:

- 1) Discuss the classification of mathematical model in detail.
- 2) In a population of birds, the proportionate birth and the proportionate death rate are both constant, being 0.45 per year and 0.65 per mear respectively. Formulate a model of the population and discuss its long term behaviour.
- 3) Form the differential equation of the free damped motion in the case of Mass Spring-Dashpot and discuss (i) over damped (ii) critically damped cases.
- 4) A radio active sustance disintegrates at a rate proportional to its mass, when its mass is 10 mgm the rate of disintegration is 0.051 mgm per day. How long will it take for the mass to be reduced from 10 mgm to 5 mgm?
- 5) Discuss the modelling of electric circuits (LCR).

